

# MODES IN LIGHT WAVE PROPAGATING IN SEMICONDUCTOR LASER

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## Abstract

The study of semiconductor laser based on an analogy of the Schrödinger equation and an equation describing light wave propagation in nonhomogeneous medium is developed. The active region of semiconductor laser is considered as optical waveguide confining the electromagnetic field in the cross-section  $(x, y)$  and allowing waveguide propagation along the laser resonator  $(z)$ . The mode structure is investigated taking into account the transversal and what is the important part of the suggested consideration longitudinal nonhomogeneity of the optical waveguide. It is shown that the Gaussian modes in this case correspond to spatial squeezing and correlation. Spatially squeezed two-mode structure of nonhomogeneous optical waveguide is given explicitly. Distribution of light among the laser discrete modes is presented. Properties of the spatially squeezed two-mode field are described. The analog of Franck-Condon principle for finding the maxima of the distribution function and the analog of Ramsauer effect for control of spatial distribution of laser emission are discussed.

## 1 Introduction

The aim of the talk is to study the possible modes of the electromagnetic field propagating in a semiconductor laser taking into account nonhomogeneous longitudinal structure of the medium in the optical waveguide of the laser. The Gaussian modes in such structures may demonstrate a squeezing phenomenon in the electromagnetic field distribution in transversal cross-section of the semiconductor laser. We will consider the light propagation along the active layer of the semiconductor laser understanding that the refractive index of the media of this layer has such dependence on the transversal coordinates  $x$  and  $y$  that provides the waveguiding conditions confining the electromagnetic field in the transversal section of the laser waveguide. We also assume that the value of the longitudinal coordinate  $z$  which varies along the laser resonator axis influences the refractive index. Our goal is to show that the refractive index dependence on the longitudinal coordinate  $z$  produces the change of the widths of the Gaussian electromagnetic field beam that may influence the far field distribution. The equation for propagating field of the fixed frequency  $\omega$  in paraxial approximation is derived from the Helmholtz equation

$$\Delta\psi + k^2(x, y, z)\psi = 0 \quad (1)$$

for which the structure of the field looks as the amplitude  $A$  which has slow dependence on the longitudinal coordinate  $z$  and the fast oscillating exponential. Due to this the equation for the

amplitude  $A(x, y, z)$  is reduced to the Schrödinger-like equation (Fock-Leontovich approximation [1], or parabolic approximation)

$$i\frac{\partial A}{\partial z} = -\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)A + k^2(x, y, z)A \quad (2)$$

in which formally the coordinate  $z$  plays the role of time and the refractive index plays the role of the potential energy. If there is no dependence of refractive index on the coordinate  $z$  the equation (2) is equivalent to the Schrödinger equation for the stationary states which in case of waveguide describe the possible modes of electromagnetic field propagating along the laser resonator. If the  $z$ -dependence of refractive index takes place the structure of the modes changes essentially. For optical fibers this phenomenon has been discussed, for example, in [2]. Our aim is to consider this phenomenon for the semiconductor laser waveguide since the physical picture of the light propagation in semiconductor laser has its own specifics and applications.

## 2 Spatial Squeezing of Semiconductor Laser Beam

As a model we will suppose that the refractive index has parabolic profile in transversal coordinates with  $z$ -dependences of the formal "frequency"-coefficients in parabolic terms  $\omega_x^2(z)x^2/2$  and  $\omega_y^2(z)y^2/2$ . We will apply the result known in the theory of nonstationary quantum oscillator to the case of optical waveguide of semiconductor laser with the parabolic refractive index of the active layer. It was shown [3] that the width of the Gaussian packet describing at the initial moment the ground state of quantum oscillator due to the time dependence of the frequency has the following value at  $t$ -moment

$$(\Delta x)^2 = \frac{|e|^2}{2} \left( \frac{\hbar}{m\omega} \right), \quad (3)$$

where the function  $e$  satisfies the equation

$$\ddot{e} + \omega^2 e = 0, \quad (4)$$

$$e(0) = 1, \quad (5)$$

$$\dot{e}(0) = i\omega(0). \quad (6)$$

Dots mean time-derivatives. We can use the complete analogy of our equation for the light propagation with Schrödinger equation for the nonstationary oscillator. Due to this analogy the following conclusion may be made for the influence of  $z$ -dependence of the refractive index on the behaviour of the fundamental mode. Its width in transversal coordinate  $x$  has the form

$$(\Delta x)^2 = |e_x|^2 (\Delta x_0)^2 \quad (7)$$

and in transversal coordinate  $y$  it has the form

$$(\Delta y)^2 = |e_y|^2 (\Delta y_0)^2, \quad (8)$$

where the functions  $e_x(z)$  and  $e_y(z)$  satisfy the equations

$$e_x''(z) + \omega_x^2(z)e_x(z) = 0, \quad (9)$$

$$e_y''(z) + \omega_y^2(z)e_y(z) = 0 \quad (10)$$

and the initial conditions

$$e_x(0) = 1, \quad e'_x(0) = i\omega_x(0), \quad (11)$$

$$e_y(0) = 1, \quad e'_y(0) = i\omega_y(0), \quad (12)$$

where  $z = 0$  is the coordinate of the left mirror of the laser resonator. The formulae (7) and (8) are given in dimensionless variables. It is known that the functions  $e_x$  and  $e_y$  may become less than 1 for appropriate  $z$ -dependence. In our case it means that for appropriate refractive index we may obtain squeezing of the light spot on the right mirror of laser resonator in comparison with the width of the field distribution on the left mirror in both transversal directions. Opposite phenomenon also may be present. It means that changing somehow the media properties along the resonator axis we could form the far field distribution. In particular there may exist spatial squeezing in the light beam propagating in semiconductor laser.

### 3 Spatially Squeezed Two-Mode Structure in the Optical Waveguide of Semiconductor Laser

We have discussed above the spatial squeezing for Gaussian modes, because the squeezing for modes which differ from fundamental one are described by the same formulae (7) and (8). Now we will consider the spatial squeezing in discrete modes of semiconductor laser using the same analogy with time-dependent oscillator. In generic case these discrete modes  $\psi_{nm}$  are described by Hermite polynomials of two variables. In our case it may be shown that the widths of light spots on the right mirror of the laser resonator may be given by formulae

$$(\Delta x)^2 = (\Delta x_0)^2 (2n + 1) |e_x|^2, \quad n = 0, 1, 2, \dots \quad (13)$$

and

$$(\Delta y)^2 = (\Delta y_0)^2 (2m + 1) |e_y|^2, \quad m = 0, 1, 2, \dots \quad (14)$$

We see that the width in both direction may be essentially reduced by choosing an appropriate  $z$ -dependence of refractive index. This spatial squeezing may be achieved both for Gaussian modes and discrete modes simultaneously. The appropriate  $z$ -dependence of the refractive index corresponds to such behaviour of the function  $e_x$  and  $e_y$  for which their moduli become much less than 1 on the right mirror. In these cases we have the spectral squeezing of the light patterns related to the discrete mode structure of the semiconductor laser beam.

### 4 Analogs of Franck-Condon Principle and Ramsauer Effect for Semiconductor Laser Nonhomogeneous along the Optical Axis

Since we have established that for the field in the semiconductor laser the formal role of potential in Schrödinger equation is played by the dielectric constant, while the role of time is played by the coordinate along the cavity axis [4], [5], [6] we have a complete analogy of the character of a

stepwise change in the properties of the active region along the laser optical axis to the character of change in vibrational motion of nuclei after the electron transition in a polyatomic molecule. In order to use this analogy, we will exploit existing results in quantum mechanics to explain the various operating conditions of a laser emphasizing that the role of the field is played by the wave function  $\Psi$ . In quantum mechanics, bound states exist when the probability density  $|\Psi|^2$  diminishes exponentially away from the potential energy minimum. For a semiconductor laser, these bound states with quantum energy levels correspond to mode solutions of the wave equation. Hence, any conclusion regarding the transitions between energy levels due to changes in a potential over time may be reformulated analogously for the case of light redistribution among the modes when two (or more) different parts of the active region are joined corresponding to inhomogeneity of the dielectric constant along the cavity axis. A continuous junction (slow change of refractive index along the cavity) is possible together with stepwise junction (a step in refractive index profile in the vicinity of the junction). In quantum mechanics of nonstationary potential the Schrödinger equation is solved in some cases for varying potential, but such cases are few and they include a specific dependence of the potential on the coordinates (see [7]). For time irregular changes in potential, in quantum mechanics the behavior of a system for the case of polyatomic molecules is predicted by the Franck-Condon principle for potentials arbitrary depending on the coordinates. Hence, this formulation of the analog of the Franck-Condon principle for semiconductor laser operation is of interest, since the qualitative predictions obtained on the basis of this principle are independent of models describing the dielectric constant of the laser active region [8].

For taking into account the nonhomogeneities of heterojunction laser along the resonator we proposed in [4], [5] the model in which the active region was considered as a set of several optical waveguides with different refractive indices connected to each other in such a way that the coupling between the separate sections of the active region was achieved with the help of mode coupling coefficients  $C_{nm}$ . Let us consider the laser consisting of two end-joined active regions (each one homogeneous along the optical axis). So, we consider in fact two connected lasers each having their own refractive index described by their own "potential curve". Each of these connected lasers is described by its own "potential well" with the "energy levels" corresponding to the discrete laser modes. If both potential wells are plotted in the same figure the analog of the Franck-Condon principle can predict which mode in the second laser (the second part of the laser) will be excited with the maximal probability, when the field energy was contained in a given mode of the first laser (the first part of the laser). This analog of the Franck-Condon principle produces a qualitative prediction for finding the maximum of the square of the modulus of the mode coupling coefficient of two lasers (two parts of the laser)

$$C_{nm} = \int Y_n^*(y) Y_m(y) dy, \quad (15)$$

i.e., an analog of the Franck-Condon factor

$$W_{nm} = |C_{nm}|^2 \quad (16)$$

for diatomic molecules (the overlap integral of the vibration wave functions before and after the electron transition). This factor describes the portion of energy contained in  $n$ -mode of the second laser (the second part of the laser) if all energy of the first laser (the first part of the laser) was

concentrated in its  $m$ - mode. Let us note that this rule is valid in a semiconductor laser when the imaginary part of the dielectric constant of its optical waveguide does not significantly change the spectrum of mode "levels" (in language of quantum mechanics, the imaginary part of the complex potential does not change the energy level spectrum, but rather is responsible only for broadening of the levels and their lifetime; however, the widths of the levels are small compared to the distance between levels).

Another analogy that would allow using results from quantum mechanics in the physics of nonhomogeneous semiconductor lasers is the analog of the Ramsauer effect [9]. Let us discuss the practical important case when distribution of dielectric constant in one transverse direction (along the  $x$ -axis) has the step-like form (in the case of heterojunction laser, the distribution of  $\epsilon(x)$  in the direction of current flow is determined by the difference between the refractive indices of the active region and the wide-band layers of the heterostructure) and in the other direction (along the  $y$ -axis) it is approximated by parabola (symmetric waveguide). The dielectric constant in the resonator is described by the formula:

$$\epsilon(y, z) = \epsilon_0 + a(z)y^2, \quad (17)$$

and the equation for electromagnetic field in the laser formally coincides with the Schrödinger equation for the harmonic oscillator with variable frequency. The analog of the Franck-Condon factor (15), (16) for our model of longitudinal nonhomogeneous laser consisting of two homogeneous parts may be presented in the form, calculated in [7]:

$$W_{nm} = \frac{n!}{m!} \sqrt{1-R} |P_{(m+n)/2}^{(m-n)/2}(\sqrt{1-R})|^2, \quad m > n, \quad (18)$$

where  $P_k^l(x)$  are the Legendre polynomials, and the parameter  $R$  can be interpreted formally as the reflection coefficient from a potential whose form is determined by the time dependence of the oscillator frequency. In our case the role of time dependent frequency is formally played by the longitudinal dependence  $a(z)$  in formula (17). It is possible to find such dependence of the "potential"  $a(z)$  to satisfy the conditions for which the equality  $R = 0$  takes place (analogue of the Ramsauer effect in quantum mechanics) and  $W_{nm} = \delta_{nm}$  (there is no redistribution of the energy of the electromagnetic field over the modes). In this case the structure of nonhomogeneity of such longitudinal nonhomogeneous laser is such that there is "transparency" in the point of connection of two lasers (two parts of the laser). With a specific dependence of  $\epsilon(y, z)$  at the connection between two part of the laser one can regulate the transparency of the connection. It is possible to use this effect for control of the output characteristics of the semiconductor laser. In the presence of dielectric insert in such a laser one can control the output characteristics of the laser by varying the characteristics of the dielectric insert [10].

The formula (18) may be considered as the mode distribution function of the laser light energy in the second laser (the second part of the laser) between its modes with index  $m$ , ( $m = 0, 1, 2, \dots$ ) if the light in the first laser (the first part of the laser) is exactly in the mode state labeled by the index  $n$ . Such distribution function emerges if the connection of two lasers is described by the change of the discussed above "frequency" related to  $z$ -dependence of the refractive index along the laser optical waveguide. More general mode distribution function may be expressed in terms

of Hermite polynomials of two variables in complete analogy with the parametric oscillator theory given in [11],[12].

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## **SECTION 4**

### **DISTRIBUTIONS IN PHASE SPACE**

